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Tripartite entanglement-dependence of tripartite non-locality in non-inertial frames

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Abstract

The three-tangle-dependence of $S_{\max} = \max\langle S\rangle$, where S is a Svetlichny operator, is explicitly derived when one party moves with a uniform acceleration with respect to other parties in a generalized Greenberger–Horne–Zeilinger and maximally slice states. The π-tangle-dependence of $S_{\max}$ is also derived implicitly. From this dependence, we conjecture that multipartite entanglement is not the only physical resource for quantum mechanical multipartite non-locality.

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(Some figures may appear in colour only in the online journal)

After Einstein–Podolsky–Rosen's seminal paper [1], the unusual properties of quantum correlations became a fundamental issue in quantum information theories. These unusual properties become manifest if one examines the Bell inequality $\langle\mathcal{B}\rangle \leqslant 2$ [2] by making use of bipartite quantum states. If this inequality is violated, then the non-locality of quantum mechanics is guaranteed. As Gisin [3] showed, the Bell-type Clauser–Horner–Shimony–Holt [4] inequality is violated for all pure entangled two-qubit states. This implies that quantum mechanics really exhibits non-local correlations. More importantly, the amount of violation $\langle\mathcal{B}\rangle - 2$ increases when the two-qubit state is entangled more and more. This fact implies that the origin of non-local correlations in quantum mechanics is an entanglement of quantum states. This remarkable fact can be used to implement the quantum cryptography [5].

Although the relationship between non-locality and entanglement is manifest to a great extent in a two-qubit system, it is not as straightforward exploring this relationship in a multipartite system. Recently, however, understanding in this field has been slightly enhanced, especially for a three-qubit system. In [6], the relationship between a Svetlichny inequality [7], a Bell-type inequality in a tripartite system, and tripartite residual entanglement called a three-tangle [8] was examined by using the generalized Greenberger–Horne–Zeilinger (GHZ) states $|\psi_g\rangle$ [9] and the maximally slice (MS) states $|\psi_s\rangle$ [10] defined as

$$|\psi_g\rangle = \cos\theta_1|000\rangle + \sin\theta_1|111\rangle \tag{1}$$
$$|\psi_s\rangle = \frac{1}{\sqrt{2}}[|000\rangle + |11\rangle\{\cos\theta_3|0\rangle + \sin\theta_3|1\rangle\}].$$

The most remarkable fact found in [6] is that the τ(three-tangle)-dependence of $S_{\max}$, the upper bound of expectation value of the Svetlichny operator, for $|\psi_g\rangle$ is

$$S_{\max}(\psi_g) = \begin{cases} 4\sqrt{1-\tau} & \tau \leqslant 1/3 \\ 4\sqrt{2\tau} & \tau \geqslant 1/3. \end{cases} \tag{2}$$

Since the Svetlichny inequality is $\langle S\rangle \leqslant 4$, whose violation guarantees non-local correlations, equation (2) shows that $|\psi_g\rangle$ really exhibits non-local correlations in the region $\tau > 1/2$. Unlike two-qubit states, however, $S_{\max}$ exhibits decreasing behaviour when $\tau \leqslant 1/3$. This fact suggests strongly that the quantum entanglement is not the only resource for multipartite non-locality. Finding the other resources which are responsible for the non-local properties of quantum mechanics, seems to be a very important issue.

The purpose of this paper is to examine the relationship between tripartite entanglement and $S_{\max}$ in non-inertial frames. Since entanglement and non-locality are the two most important concepts in quantum mechanics, the relationship between them is at the heart of quantum mechanics. Recently, the relations of several three-qubit pure states were derived in a non-relativistic framework [6, 11]. The original purpose of this paper was to extend these relations to the relativistic framework. Since, furthermore, the analysis of non-inertial frames generally transforms a pure state into a mixed state due to the Unruh decoherence effect [12, 13], as a by-product of this, one can derive the relationship between tripartite entanglement and $S_{\max}$ for the various mixed states in this paper.

Although a similar issue was considered recently in [14], the authors in that study chose only π-tangle [15] as a tripartite entanglement measure. However, the explicit π-tangle-dependence of $S_{\max}$ was not derived in [14]. Furthermore, as far as we know, there are two different tripartite entanglement measures: three-tangle [8] and π-tangle [15]. Unlike the π-tangle, the three-tangle has its own historical background. In fact, it exactly coincides with the modulus of a Cayley hyperdeterminant [16, 17], which was constructed long ago. It is also a polynomial invariant under the local $SL(2, \mathbb{C})$ transformation [18, 19]. Thus, it seems to be more meaningful to derive the three-tangle-dependence of the $S_{\max}$ explicitly.

However, the calculation of the three-tangle for three-qubit mixed states is much more difficult than that of the π-tangle. Since the three-tangle for a mixed state ρ is defined by the convex roof method [20, 21]

$$\tau(\rho) = \min \sum_j P_j \tau(\rho_j), \tag{3}$$

where the minimum is taken over all possible ensembles of pure states ρ_j with $0 \leqslant P_j \leqslant 1$, the explicit computation of the three-tangle needs to derive an optimal decomposition of the given mixed state ρ. It causes difficulties in the analytic computation of the three-tangle. Recently, however, various techniques [22–27] were developed to overcome these difficulties. However, it is still very important to compute the three-tangle analytically for high-rank mixed states except in very rare cases. Fortunately, the mixture derived in this paper is only rank-2. Thus, it is possible to compute the three-tangle analytically, using various techniques developed in [22–27]. In this paper we use these techniques to derive the relations between the three-tangle and the $S_{\max}$ in non-inertial frames.

Now, we assume that Alice, Bob and Charlie initially share the generalized fermionic GHZ state $|\psi_g\rangle_{ABC}$ or the MS state $|\psi_s\rangle_{ABC}$. We also assume that after sharing his own qubit, Charlie moves with respect to Alice and Bob with a uniform acceleration a. Then, Charlie's vacuum and the one-particle states $|0\rangle_M$ and $|1\rangle_M$, where the subscript M stands for Minkowski, are transformed into [28]

$$|0\rangle_M \rightarrow \cos r|0\rangle_I|0\rangle_{II} + \sin r|1\rangle_I|1\rangle_{II} \tag{4}$$
$$|1\rangle_M \rightarrow |1\rangle_I|0\rangle_{II},$$

where the parameter r is defined by

$$\cos r = \frac{1}{\sqrt{1+\exp(-2\pi\omega c/a)}}, \tag{5}$$

and c is the speed of light, and ω is the central frequency of the fermion wave packet[1]. Thus, $r = 0$ when $a = 0$ and $r = \pi/4$ when $a = \infty$. In equation (4), $|n\rangle_I$ and $|n\rangle_{II}$ ($n = 0, 1$) are the mode decompositions in the two causally disconnected regions in Rindler space. Therefore, equation (4) implies that the physical information initially formed in region I is leaked into region II, which is a key element of the Unruh effect [12, 13].

Before we discuss the relationship between the Svetlichny inequality and tripartite entanglement, we should comment that the superselection rule (SSR) of the fermion fields [29] does not allow $|\psi_g\rangle_{ABC}$ and $|\psi_s\rangle_{ABC}$ as fermion states. This can be easily confirmed by the fact that $|\psi_g\rangle\langle\psi_g|$ and $|\psi_s\rangle\langle\psi_s|$ do not commute with $(-1)^{\hat{F}} = \mathrm{diag}\{1, -1, -1, 1, -1, 1, 1, -1\}$, where $\hat{F}$ is the fermion number operator [30]. Recently, the SSR and other subtle issues for fermion fields were discussed in the context of relativistic quantum information theories [31–33]. Furthermore, as discussed in [30], this SSR constraint also modifies the definition of the three-tangle for mixed states because the optimal decompositions should also obey the SSR constraint. If, therefore, the SSR is taken into account, equation (3) yields merely the lower bound of the three-tangle.

Despite this, we ignore the restriction generated by the SSR in this paper. The main reason for this is that as Weinberg commented [29], it is always possible to enlarge the symmetry group to a new one that lacks the SSR. Thus, it is possible to remove the SSR restriction by extending the symmetry group appropriately.

Using equation (4) one can easily show that Charlie's acceleration makes $|\psi\rangle_{ABC}$ to be

$$|\psi\rangle_{ABC} \rightarrow [\cos\theta_1 \cos r|000\rangle + \sin\theta_1|111\rangle] \otimes |0\rangle_{II} + \cos\theta_1 \sin r|001\rangle \otimes |1\rangle_{II}, \tag{6}$$

where $|\alpha\beta\gamma\rangle \equiv |\alpha\beta\rangle_{AB}^M \otimes |\gamma\rangle_I$. Since $|\psi\rangle_{II}$ is a physically inaccessible state from region I, it is reasonable to take a partial trace over II to average it out. Then, the remaining quantum state becomes the following mixed state:

$$\rho_{ABI} = \cos^2\theta_1 \cos^2 r|000\rangle\langle 000| + \cos^2\theta_1 \sin^2 r|001\rangle\langle 001| + \sin^2\theta_1|111\rangle\langle 111|$$
$$+ \sin\theta_1 \cos\theta_1 \cos r\{|000\rangle\langle 111| + |111\rangle\langle 000|\}. \tag{7}$$

The maximum of the expectation value of the Svetlichny operator, $S_{\max}$, for ρ_{ABI} was explicitly derived in [14], and the final expression can be written as

$$S_{\max} = 4\max[|2\cos^2\theta_1\cos^2 r - 1|, \sqrt{2}|\sin 2\theta_1|\cos r]. \tag{8}$$

When $a = 0$, equation (8) reduces to $S_{\max} = 4\max[|2\cos^2\theta_1 - 1|, \sqrt{2}|\sin 2\theta_1|]$, which ensures that the violation of the Svetlichny inequality arises when $\pi/8 < \theta_1 < 3\pi/8$ in a region

[1] For the bosonic state equation (4) is changed into

$$|0\rangle_M \rightarrow \frac{1}{\cosh r}\sum_{n=0}^{\infty}\tanh^n r|n\rangle_I|n\rangle_{II} \qquad |1\rangle_M \rightarrow \frac{1}{\cosh^2 r}\sum_{n=0}^{\infty}\tanh^n r\sqrt{n+1}|n+1\rangle_I|n\rangle_{II},$$

where

$$\cosh r = \frac{1}{\sqrt{1-\exp(-2\pi\omega c/a)}}.$$

$0 \leqslant \theta_1 \leqslant \pi/2$. When $a = \infty$, equation (8) reduces to $S_{\max} = 4\max[1 - \cos^2\theta_1, \sin 2\theta_1]$, which shows that there is no violation of the Svetlichny inequality.

Now, we discuss the tripartite entanglement of ρ_{ABI} given in equation (7). The computation of its π-tangle is straightforward and the final expression becomes

$$\pi_{\mathrm{GGHZ}} = \frac{2 + \cos^2 r}{3} \sin^2 2\theta_1 + \frac{1}{3} \cos^4\theta_1 \sin^2 2r. \tag{9}$$

When, therefore, $a = 0$, π_{GGHZ} becomes $\sin^2 2\theta_1$, which shows that $|\psi_g\rangle$ is maximally entangled at $\theta_1 = \pi/4$ and non-entangled at $\theta_1 = 0$ and $\pi/2$. When $a = \infty$, equation (9) reduces to $\pi_{\mathrm{GGHZ}} = (5/6)\sin^2 2\theta_1 + (1/3)\cos^4\theta_1$, which is maximized by $25/27 \sim 0.926$ at $\theta_1 = \sin^{-1}(2/3)$ and minimized by zero at $\theta_1 = \pi/2$. The nonvanishing tripartite entanglement at $a \to \infty$ limit was discussed in [34]. This property differs crucially from the bosonic bipartite entanglement, which completely vanishes at $a \to \infty$ limit [35].

In order to compute the three-tangle it is convenient to use the spectral decomposition of ρ_{ABI}, whose expression is

$$\rho_{ABI} = p|\mathrm{GHZ}\rangle\langle\mathrm{GHZ}| + (1 - p)|001\rangle\langle 001|, \tag{10}$$

where $|\mathrm{GHZ}\rangle = a|000\rangle + b|111\rangle$ with

$$p = \cos^2\theta_1 \cos^2 r + \sin^2\theta_1 \qquad a = \frac{\cos\theta_1 \cos r}{\sqrt{\sin^2\theta_1 + \cos^2\theta_1 \cos^2 r}}$$
$$b = \frac{\sin\theta_1}{\sqrt{\sin^2\theta_1 + \cos^2\theta_1 \cos^2 r}}. \tag{11}$$

In order to derive the optimal decomposition we define

$$|Z(\phi)\rangle = \sqrt{p}|\mathrm{GHZ}\rangle + \mathrm{e}^{\mathrm{i}\phi}\sqrt{1 - p}|001\rangle. \tag{12}$$

This has several interesting properties. First, ρ_{ABI} given in equation (10) can be written as

$$\rho_{ABI} = \tfrac{1}{2}[|Z(\phi)\rangle\langle Z(\phi)| + |Z(\phi + \pi)\rangle\langle Z(\phi + \pi)|]. \tag{13}$$

Secondly, the three-tangle of $|Z(\phi)\rangle$ is independent of ϕ as $\tau_Z = 4p^2a^2b^2$. If, therefore, equation (13) is an optimal decomposition, the three-tangle of ρ_{ABI} is also $\tau_{ABI} = 4p^2a^2b^2$. Since τ_{ABI} is convex with respect to p, this fact guarantees that equation (13) is really the optimal decomposition for ρ_{ABI}. Using equation (11) it is easy to show

$$\tau_{ABI} = \sin^2 2\theta_1 \cos^2 r. \tag{14}$$

Therefore, combining equation (8) and equation (14) we get the explicit three-tangle-dependence of $S_{\max}$ as following:

$$S_{\max} = 4\max[\sqrt{\cos^2 r - \tau_{ABI}}\cos r - \sin^2 r, \sqrt{2\tau_{ABI}}]. \tag{15}$$

When $a = 0$, it is easy to show that equation (2) is reproduced.

In figure 1(a) we plot the three-tangle-dependence of π-tangle when $a = 0, 2\omega c, 5\omega c$, and $10\omega c$. As expected from the fact that these are two different tripartite entanglement measures, π-tangle is monotonous with respect to the three-tangle. Figure 1(*a*) also shows that regardless of acceleration, the a π-tangle is larger than the three-tangle, which was conjectured in [15, 26].

Figure 1(*b*) and figure 1(*c*) show the tripartite entanglement-dependence of $S_{\max}$. As figure 1(*b*) exhibits, the violation of the Svetlichny inequality, i.e. $S_{\max} > 4$, occurs when $\pi_{ABI} > \pi_*$, where π_* increases with increasing as. The critical value π_* is given in table 1 for various a. As table 1 shows, π_* approaches 1 at the $a \to \infty$ limit, which implies that there is no violation of the Svetlichny inequality at this limit. Figure 1(*c*) is a plot of the τ_{ABI}-dependence of $S_{\max}$ for various a. As figure 1(*c*) exhibits, the violation of the Svetlichny inequality occurs

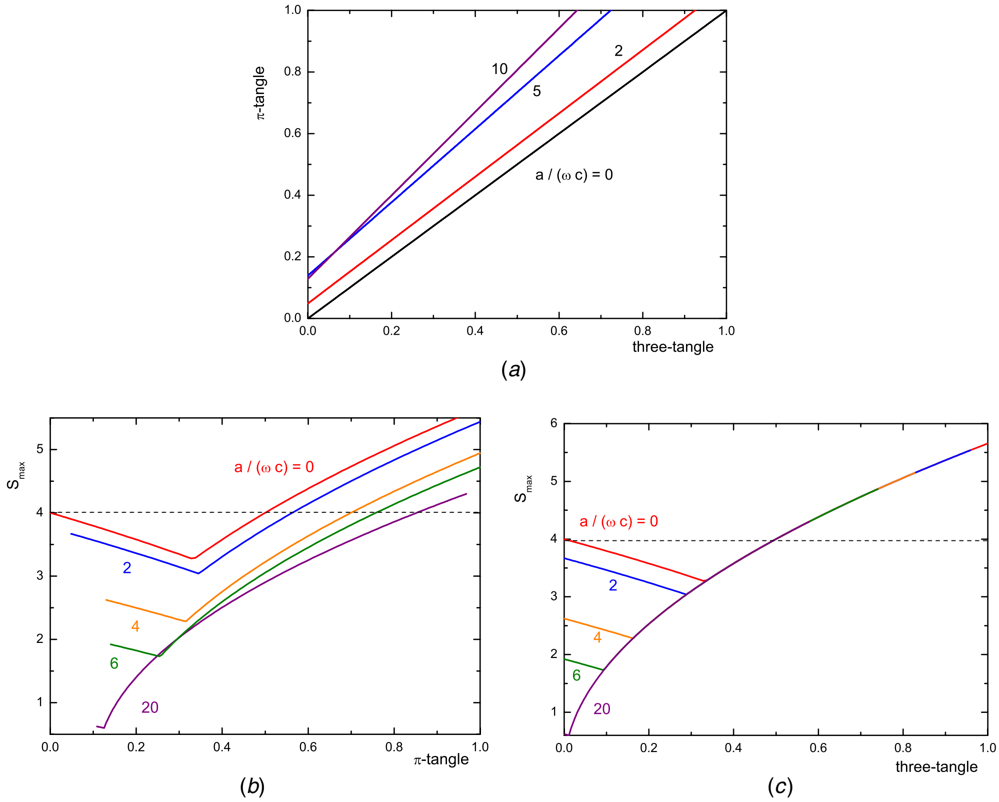

Figure 1. In (a) we plot the π-tangle (9) versus the three-tangle (14). The π-tangle exhibits monotonous behaviour with respect to the three-tangle. This fact is plausible because these tangles are two different measures for tripartite entanglement. In (b) and (c) we plot the tripartite entanglement-dependence of $S_{\max}$. These figures show that $S_{\max}$ exhibits decreasing behaviour in the small entanglement region. This fact seems to imply that entanglement is not a unique physical resource for quantum mechanical non-locality.

Table 1. Acceleration dependence of π_* and τ_*.

$a/\omega c$	0	2	4	6	8	10	100	∞
π_*	0.50	0.563	0.70	0.757	0.787	0.806	0.901	1
τ_*	1	0.959	0.828	0.740	0.687	0.652	0.566	0.5

when $\tau_{ABI} > 0.5$ for all a. The maximum of the three-tangle, i.e. τ_*, is dependent on Charlie's acceleration, a. As table 1 shows, τ_* exhibits a decreasing behaviour with increasing a, and eventually approaches 0.5 in $a \to \infty$ limit. This fact also indicates that the state shared initially by Alice, Bob and Charlie cannot have a non-local property in Charlie's infinite acceleration although it has nonzero tripartite entanglement.

If Alice, Bob and Charlie initially share the MS state $|\psi_s\rangle_{ABC}$, Charlie's acceleration changes $|\psi_s\rangle_{ABC}$ into

$$\sigma_{ABI} = \frac{1}{2}[\cos^2 r|000\rangle\langle 000| + \sin^2 r|001\rangle\langle 001| + \cos^2\theta_3 \cos^2 r|110\rangle\langle 110| \\ +(\sin^2\theta_3 + \cos^2\theta_3 \sin^2 r)|111\rangle\langle 111| + \cos\theta_3 \cos^2 r\{|000\rangle\langle 110| + |110\rangle\langle 000|\}$$

$$+ \sin\theta_3 \cos r\{|000\rangle\langle 111| + |111\rangle\langle 000|\} + \cos\theta_3 \sin^2 r\{|001\rangle\langle 111| + |111\rangle\langle 001|\} + \sin\theta_3\cos\theta_3\cos r\{|110\rangle\langle 111| + |111\rangle\langle 110|\}]. \tag{16}$$

The maximum of $\langle S\rangle = \mathrm{tr}[\sigma_{ABI} S]$ was explicitly computed in [14], which has the form

$$S_{\max} = 4[\cos^2\theta_3 \cos^2 2r + 2\sin^2\theta_3 \cos^2 r]^{1/2}. \tag{17}$$

Thus, $S_{\max} \geqslant 4$ for $a = 0$ and $S_{\max} \leqslant 4$ for $a = \infty$.

The π-tangle for σ_{ABI} can be computed straightforwardly and its final expression is

$$\pi_{MS} = \frac{1}{3}[\sin^2\theta_3(2 + \cos^2 r) + \sin^2 r \cos^2 r(1 + \cos^2\theta_3)^2]. \tag{18}$$

In order to compute the three-tangle for σ_{ABI}, we express σ_{ABI} in terms of eigenvectors as following:

$$\sigma_{ABI} = \Lambda_+|\Psi_+\rangle\langle\Psi_+| + \Lambda_-|\Psi_-\rangle\langle\Psi_-| \tag{19}$$

where

$$\Lambda_\pm = \frac{1 \pm \sqrt{\Delta}}{2} \qquad |\Psi_\pm\rangle = \frac{1}{\mathcal{N}_\pm}\Big[X_\pm|000\rangle + Y_\pm|001\rangle + Z_\pm|110\rangle + W_\pm|111\rangle\Big]. \tag{20}$$

In equation (20) $\Delta = \cos^2\theta_3 + \cos^2 r[\sin^2\theta_3 - \sin^2 r(1 + \cos^2\theta_3)^2]$ and

$$X_\pm = \cos r(\mu \pm \sqrt{\Delta}) \qquad Y_+ = Y_- = \sin\theta_3\cos\theta_3\sin^2 r \qquad Z_\pm = \cos\theta_3 X_\pm \qquad W_\pm = \sin\theta_3(\cos^2 r \pm \sqrt{\Delta}) \tag{21}$$

with $\mu = \cos^2 r - \sin^2 r\cos^2\theta_3$. The normalization constants $\mathcal{N}_\pm$ are

$$\mathcal{N}_\pm^2 = X_\pm^2 + Y_\pm^2 + Z_\pm^2 + W_\pm^2 = \pm 2\sqrt{\Delta}[(1 + \mu)(\cos^2 r \pm \sqrt{\Delta}) - \sin^2 r\cos^2 r\cos^2\theta_3(1 + \cos^2\theta_3)]. \tag{22}$$

Then, it is easy to show $\langle\Psi_+|\Psi_-\rangle = 0$. We now define

$$|\Phi_\pm(\varphi)\rangle = \sqrt{\Lambda_+}|\Psi_+\rangle \pm \mathrm{e}^{\mathrm{i}\varphi}\sqrt{\Lambda_-}|\Psi_-\rangle. \tag{23}$$

Then, σ_{ABI} can be written as

$$\sigma_{ABI} = \frac{1}{2}|\Phi_+(\varphi)\rangle\langle\Phi_+(\varphi)| + \frac{1}{2}|\Phi_-(\varphi)\rangle\langle\Phi_-(\varphi)|. \tag{24}$$

The three-tangle $\tau(\Phi_\pm)$ for $|\Phi_\pm(\varphi)\rangle$ is

$$\tau(\Phi_\pm) = 4|\tilde{X}_\pm\tilde{W}_\pm - \tilde{Y}_\pm\tilde{Z}_\pm|^2 \tag{25}$$

where $\tilde{G}_\pm = \sqrt{\Lambda_+}G_+/\mathcal{N}_+ \pm \mathrm{e}^{\mathrm{i}\varphi}\sqrt{\Lambda_-}G_-/\mathcal{N}_-$ with $G = X, Y, Z$, or W. Thus, if equation (24) is an optimal decomposition for σ_{ABI}, the three-tangle becomes

$$\tau(\sigma_{ABI}) = \frac{4\Lambda_+^2}{\mathcal{N}_+^4}(X_+W_+ - Y_+Z_+)^2 + \frac{4\Lambda_-^2}{\mathcal{N}_-^4}(X_-W_- - Y_-Z_-)^2 + \frac{4\Lambda_+\Lambda_-}{\mathcal{N}_+^2\mathcal{N}_-^2}\{(X_+W_- + X_-W_+) - (Y_+Z_- + Y_-Z_+)\}^2 + \frac{8\Lambda_+\Lambda_-}{\mathcal{N}_+^2\mathcal{N}_-^2}(X_+W_+ - Y_+Z_+)(X_-W_- - Y_-Z_-)\cos 2\varphi. \tag{26}$$

Since $(X_+W_+ - Y_+Z_+)(X_-W_- - Y_-Z_-) = \cos^2 r\sin^4 r\cos^4\theta_3\sin^6\theta_3 \geqslant 0$, we have to choose $\varphi = \pi/2$ to minimize $\tau(\sigma_{ABI})$. Then, $\tau(\sigma_{ABI})$ simply reduces to

$$\tau(\sigma_{ABI}) = \cos^2 r\sin^2\theta_3. \tag{27}$$

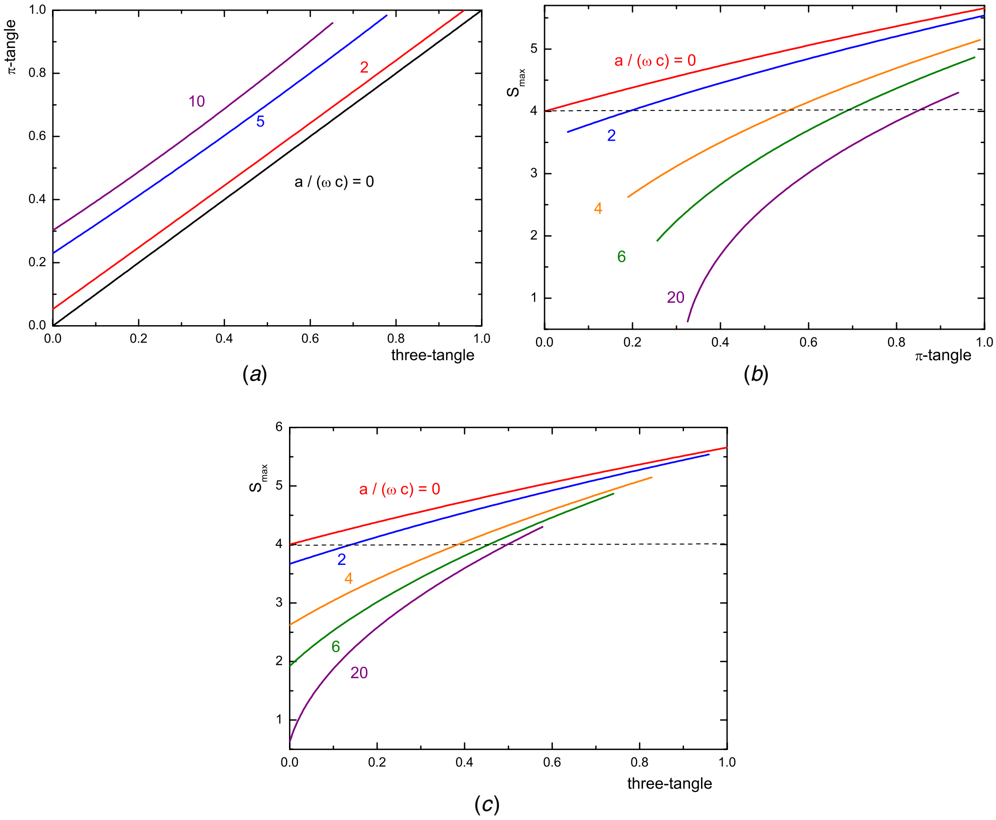

Figure 2. In (a) we plot the π-tangle (18) versus the three-tangle (27). As figure 1(a) the π-tangle exhibits monotonous behaviour with respect to the three-tangle. Regardless of acceleration a the π-tangle is larger than the three-tangle, which might be true generally, as conjectured in [15, 26]. In (b) and (c) we plot the tripartite entanglement-dependence of $S_{\max}$. Unlike figure 1(b) and figure 1(c) the decreasing behaviour of $S_{\max}$ in small entanglement region disappears.

It is interesting to note that the three-tangle is much simpler than the π-tangle. From equation (17) and equation (27) one can derive the three-tangle-dependence of $S_{\max}$, which is

$$S_{\max} = 4\sqrt{\cos^2 2r + (5 - 4\cos^2 r - \tan^2 r)\tau(\sigma_{ABI})}. \tag{28}$$

When $a = 0$, equation (28) reduces to $S_{\max} = 4\sqrt{1 + \tau(\sigma_{ABI})}$. Thus, the violation of the Svetlichny inequality occurs for all nonzero three-tangles. When $a = \infty$, equation (28) reduces to $S_{\max} = 4\sqrt{2\tau(\sigma_{ABI})}$, which implies that the violation of the Svetlichny inequality occurs when $\tau(\sigma_{ABI}) > 1/2$.

In figure 2(a) we plot the three-tangle-dependence of π-tangle for σ_{ABI} when $a = 0$, $2\omega c$, $5\omega c$, and $10\omega c$. As in figure 1(a) the π-tangle (18) is monotonous with respect to the three-tangle (27). Figure 2(a) also indicates that π-tangle is in general larger than the three-tangle. In figure 2(b) and figure 2(c) we plot the tripartite entanglement-dependence of $S_{\max}$. Unlike figure 1(b) and figure 1(c) there is no decreasing behaviour of $S_{\max}$ in these figures. From figure 2(b) and figure 2(c) we know that π_c and τ_c increase with increasing a if the violation of the Svetlichny inequality occurs when $\pi_{\mathrm{MS}} > \pi_c$ and $\tau(\sigma_{ABI}) > \tau_c$. These critical values are given in table 2 for various a. Table 2 shows that $\pi_c \to 1$ and $\tau_c \to 0.5$ in the infinite acceleration limit.

Table 2. Acceleration dependence of π_c and τ_c.

$a/\omega c$	0	2	4	6	8	10	100
π_c	0	0.191	0.250	0.685	0.746	0.780	0.901
τ_c	0	0.142	0.385	0.456	0.479	0.488	0.5

If Bob moves, instead of Charlie, with a uniform acceleration, the initial state $|\psi\rangle_{ABC}$ is transformed into

$$\sigma_{AIC} = \frac{1}{2}[\cos^2 r|000\rangle\langle 000| + \sin^2 r|010\rangle\langle 010| + \cos^2\theta_3|110\rangle\langle 110| + \sin^2\theta_3|111\rangle\langle 111| + \cos r\cos\theta_3\{|000\rangle\langle 110| + |110\rangle\langle 000|\} + \cos r\sin\theta_3\{|000\rangle\langle 111| + |111\rangle\langle 000|\} + \sin\theta_3\cos\theta_3\{|110\rangle\langle 111| + |111\rangle\langle 110|\}]. \tag{29}$$

The maximum of $\langle S\rangle = \mathrm{tr}[\sigma_{AIC}S]$ was given in [14], which is

$$S_{\max} = 4\cos r[\cos^2\theta_3 + 2\sin^2\theta_3]^{1/2}. \tag{30}$$

The π-tangle for σ_{AIC} can be computed straightforwardly and the final expression is

$$\tilde{\pi}_{\mathrm{MS}} = \frac{1}{3}\left[1 + \sin^2\theta_3 - \cos^2 r\cos 2\theta_3 + \sin^2 r\cos 2r + \sin^2 r\sqrt{\sin^4 r + 4\cos^2 r\cos^2\theta_3}\right]. \tag{31}$$

Using a similar method one can compute the three-tangle for σ_{AIC}, which is exactly the same as $\tau(\sigma_{ABI})$ given in equation (27). Therefore, the three-tangle-dependence of $S_{\max}$, in this case is

$$S_{\max} = 4\sqrt{\cos^2 r + \tau(\sigma_{AIC})}. \tag{32}$$

Equation (32) implies that the violation of the Svetlichny inequality arises for all nonzero $\tau(\sigma_{AIC})$ when $a = 0$. It also implies that $\tau(\sigma_{AIC}) \leqslant 1/2$ when $a \to \infty$ is the limit because $S_{\max} \leqslant 4$ in this limit.

In this paper we have examined the tripartite entanglement-dependence of $S_{\max} = \max\langle S\rangle$, where S is the Svetlichny operator, when one party moves with a uniform acceleration a with respect to other parties. If the initial tripartite state is the generalized GHZ state $|\psi_g\rangle_{ABC}$, the three-tangle-dependence of $S_{\max}$ is analytically derived in equation (15). As figure 1 shows, $S_{\max}$ exhibits decreasing behaviour in the small tripartite entanglement region while it exhibits an increasing behaviour in the large tripartite entanglement region. This fact seems to suggest that the tripartite entanglement is not the only physical resource for tripartite non-locality. If the initial state is the MS state $|\psi_s\rangle_{ABC}$, the explicit relations between $S_{\max}$ and the three-tangle are derived in equation (28) and equation (32). In this case the decreasing behaviour of $S_{\max}$ disappears as figure 2 shows. The a-dependence of the critical values π_*, τ_*, π_c, and τ_c is summarized in table 1 and table 2.

It would seem interesting to generalize our results to the tripartite bosonic cases [34]. In this case, however, it is very difficult to compute $S_{\max}$ in a non-inertial frame because the acceleration of one party transforms the qubit system at $a = 0$ into a qubit system for nonzero a. In order to analyse this issue, we should define the Svetlichny-like inequality in the qubit system.

As equations (8), (17), and (30) show, the violation of the Svetlichny inequality does not occur in the $a \to \infty$ limit [36] even if the tripartite entanglement does not completely vanish at this limit. This fact suggests that although there is some connection between the tripartite non-locality and the tripartite entanglement, the entanglement is not a unique resource for the non-locality. What then, are the other physical resources responsible for the non-locality of quantum mechanics? As far as we know, we do not yet have definite answer. We will continue to study this issue in the future.

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